

The Spy Who Logged Me!

1) $\log(4x-1) = 2$

$$10^2 = 4x-1$$

$$100 = 4x-1$$

$$101 = 4x$$

$$\frac{101}{4} = x$$

2) $\log_3(5-x) = 3$

$$3^3 = 5-x$$

$$27 = 5-x$$

$$x = 5-27$$

$$x = -22$$

3) $\log_2(x+6) + \log_2(3) = \log_2 30$

$$\log_2[(x+6)(3)] = \log_2 30$$

$$(x+6)(3) = 30$$

$$3x + 18 = 30$$

$$3x = 12$$

$$x = 4$$

4) $\log_3 x + \log_3(x-1) = \log_3(4x)$

$$\log_3[x(x-1)] = \log_3(4x)$$

$$x(x-1) = 4x$$

$$x^2 - x = 4x$$

$$x^2 - x - 4x = 0$$

$$x^2 - 5x = 0$$

$$\rightarrow x(x-5) = 0$$

$$x=0 \quad x-5=0$$

$$x=5$$

$$5) \log_6 (x+3) + \log_6 (x-2) = 1$$

$$\log_6 [(x+3)(x-2)] = 1$$

$$6^1 = (x+3)(x-2)$$

$$6 = x^2 - 2x + 3x - 6$$

$$0 = x^2 + x - 12$$

$$0 = (x+4)(x-3)$$

$$\boxed{x = -4} \text{ or } x = 3$$

inadmissible

Cannot take log of negative.

$$6.) \log_2 (x+1) - \log_2 (x-1) = 6$$

$$\log_2 \left(\frac{x+1}{x-1} \right) = 6$$

$$2^6 = \frac{x+1}{x-1}$$

$$64 = \frac{x+1}{x-1}$$

$$64(x-1) = x+1$$

$$64x - 64 = x + 1$$

$$64x - x = 1 + 64$$

$$63x = 65$$

$$x = \frac{65}{63}$$

$$\begin{aligned} 1) \quad 1 - \log(x-4) &= \log(x+5) \\ 1 &= \log(x+5) + \log(x-4) \\ 1 &= \log[(x+5)(x-4)] \end{aligned}$$

$$10^1 = (x+5)(x-4)$$

$$10 = x^2 - 4x + 5x - 20$$

$$0 = x^2 + x - 30$$

$$0 = (x+6)(x-5)$$

$$\boxed{x = -6} \text{ or } x = 5$$

inadmissible

$$ii) \quad 2^{2x} - 2^x - 6 = 0$$

$$\text{let } 2^x = y$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3 \text{ or } y = -2$$

$$2^x = 3 \text{ or } 2^x = -2$$

$$\log 2^x = \log 3$$

not possible.

$$x \log 2 = \log 3$$

$$x = \frac{\log 3}{\log 2}$$

$$\text{iii) } 3^{2x} + 2(3^x) - 15 = 0$$

$$\text{let } y = 3^x$$

$$y^2 + 2y - 15 = 0$$

$$(y+5)(y-3) = 0$$

$$y = -5 \quad y = 3$$

$$3^x = -5 \quad 3^x = 3$$

$$\text{not possible} \quad 3^x = 3^1$$

$$x = 1$$

$$\text{iv) } 10^{2x} + 5(10^x) + 4 = 0$$

$$\text{let } y = 10^x$$

$$y^2 + 5y + 4 = 0$$

$$(y+4)(y+1) = 0$$

$$y = -4 \quad y = -1$$

$$10^x = -4$$

$$10^x = -1$$

not possible

not possible

$$\text{vi) } 5^{3x} - 2(5^{2x}) - 5^x + 2 = 0$$

$$5^{2x}(5^x - 2) - 1(5^x - 2) = 0$$

$$(5^{2x} - 1)(5^x - 2) = 0$$

$$5^{2x} - 1 = 0 \quad \text{or} \quad 5^x - 2 = 0$$

$$5^{2x} = 1$$

$$5^x = 2$$

$$\log 5^{2x} = \log 1$$

$$\log 5^x = \log 2$$

$$2x \log 5 = \log 1$$

$$x \log 5 = \log 2$$

$$x = \frac{\log 1}{2 \log 5}$$

$$x = \frac{\log 2}{\log 5}$$

$$= 0$$

$$\text{or } 5^{2x} = 1$$

$$5^{2x} = 5^0$$

$$2x = 0$$

$$x = 0$$

$$\log_4 x + \log_8 x + \log_{16} x = 25$$

$$2 \log_2 x + \frac{1}{3} \log_2 x + \frac{1}{4} \log_2 x = 25$$

$$\frac{3}{2} \log_2 x + \frac{1}{3} \log_2 x + \frac{1}{4} \log_2 x = 25$$

$$\log_2 x \left(\frac{3}{2} + \frac{1}{3} + \frac{1}{4} \right) = 25$$

$$\begin{array}{l} y \\ x \\ x \\ = \log x \\ = \log x \\ = \log x \end{array} \quad \left(\begin{array}{l} \textcircled{A} \log_2 x = y \\ 2^y = x \end{array} \right.$$

$$\begin{aligned} \log 2^y &= \log x \\ y \log 2 &= \log x \\ y &= \frac{\log x}{\log 2} \end{aligned}$$

$$\log_2 x = \frac{\log x}{\log 2}$$

$$\begin{aligned} &= \frac{\log x}{\log 2} \\ &= \frac{1 \log x}{2 \log 2} \end{aligned}$$